Deformation and fracture of paper during the in-plane fracture toughness testing—Examination of the essential work of fracture method

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The "essential work of fracture" (EWF) method is applied to various machine-made papers. The deforming and fracturing processes of the paper samples during testing is analyzed by means of the thermographic observation. Plastic deformation zone appears in three ways when deep double edge notched tension specimens are strained under in-plane stress: i.e. 1. type (i)—appearing through whole the ligament in a vague manner and developing into a circular (or oval) zone even before or at the maximum load point; 2. type (ii)—appearing from notch tip and amalgamating into a circular (or oval) zone after the maximum load point; and 3. type (iii)—appearing from notch tip and not amalgamating into a circular (or oval) zone until the sheet failure. Specimens with small ligament length (L) are likely to belong to type (i), while those with large L to type (ii) & (iii). Among these three types, type (i) fulfills the original assumption of the EWF method best in terms of the complete ligament yielding before crack initiation. Thus the specific essential work of fracture determined using the linear relation of type (i) should be correct, although the estimated work is a little smaller than that from the linear relation of type (ii) & (iii).

1. Introduction

Paper strength evaluation based on fracture mechanics has been extensively investigated by many scientists for last two decades [1, 2]. Especially for "J-integral" and "essential work of fracture (EWF)" methods, standardization of the experimental method and proposal for standard testing method have started to be discussed after piling up of many experimental results [3]. Furthermore a special instrument for standard "J-integral" testing has already become commercially available [4]. However, some basic problems are unsolved yet for both methods; for example, different determination methods do not show the same value of fracture toughness using "J-integral" method and the appearance of circular plastic deformation zone has not been confirmed in the EWF method. In a previous study [5], deforming and fracturing processes during the EWF method testing were observed by means of thermography for handsheets from softwood kraft pulp to ascertain stress concentration at around notch tips, stable crack growth after maximum load point and appearance of circular plastic deformation zone before final sheet failure. However, many commercial papers are rather brittle in fracturing behavior contrasting to that of the handsheets from softwood kraft pulp.

In the present study, the EWF method, which is primarily suitable for ductile material, is applied to commercial papers, too. The deforming and fracturing processes of the paper during the testing are analyzed by means of the thermographic observation. The application of the EWF method to paper materials is discussed in detail using these results.

2. Theory

The EWF method introduced by Seth *et al.* [6, 7] is originally developed for characterizing ductile fracture by Cotterell and Reddel [8, 9]. If a deep double edge notched tension (DENT) specimen yields completely before fracture, the plastic deformation zone is almost circular on the ligament (between double notches) as shown in Fig. 1. Furthermore the work performed to fracture such a specimen (W_f) can be separated into two components:

(1) The essential work performed in the fracture process zone (W_e).
(2) The non essential work performed in the plastic deformation zone (W_p).

The essential work is proportional to the ligament length (L) if it is assumed that the specific essential work of fracture (w_e) remains constant. And the non essential work in the rest of the plastic deformation zone

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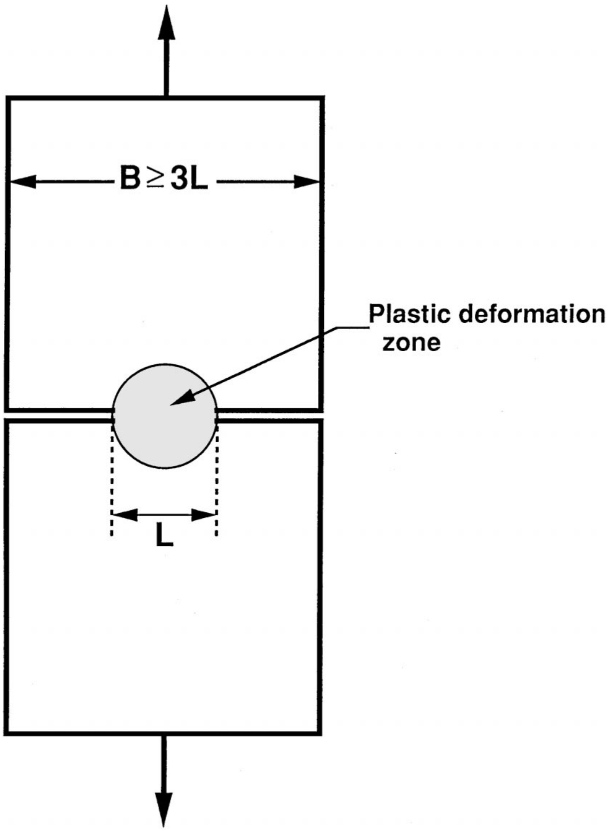

Figure 1 Deep double edge notched tension specimen for the in-plane fracture toughness testing and showing the plastic deformation zone [8, 9]. (B: specimen width, L: ligament length).

is proportional to L^2 if it is assumed that the specific non essential work of fracture (w_p) remains constant. After all, the model is represented in the simple equation as

$$W_f = W_e + W_p \tag{1}$$

$$= Ltw_e + \beta L^2 tw_p \tag{2}$$

where t is the sample thickness and β is a shape factor of the plastic deformation zone. By referring all terms to the unit surface,

$$w_f\left(= \frac{W_f}{Lt}\right) = w_e + \beta L w_p \tag{3}$$

is obtained. According to this equation, a linear relationship is expected between L and w_f. And w_e can be obtained by extrapolating to $L = 0$, considered as fracture toughness.

These relations are valid if there is complete yielding of the ligament before crack initiation and the ligament is in a state of plane stress. Thus there is an upper and lower limit to the ligament length L. The lower limit is governed by the sheet thickness (t) and is of the order $L > 5t$. In the case of paper sheet, this limit is thoroughly cleared up.

The upper limit is determined in two ways. First, if L is not small compared to the width, then the plastic zone size can be disturbed by edge effects. In order to avoid these effects, L is recommended to be kept below 1/3 of the specimen width. Secondly, L should not be larger than the size of the plastic deformation zone ahead of a notch tip. After all, it is proposed that L should be smaller than both of the two criteria:

$$L \leq \left(\frac{W}{3}, 2r_p\right)$$

where r_p is the plastic zone size parameter which is given by linear elastic fracture mechanics.

3. Experimental

3.1. Material

Various machine-made papers (sack paper from unbleached kraft pulp, machine glazed paper, newsprint paper, and filter paper) were employed in this study. Their basic properties are given in Table I.

Size of DENT specimens, notching and specimen setting to a pair of clamps are the same as those described in the previous paper [5]. Ligament length (L), distance between double notches, was 1/3 of the specimen width which is in the range of 1 mm $\leq L \leq$ 21 mm.

Experimental plots between work of fracture and ligament lengths were linear in most part of the range. The specific essential work of fracture (w_e) obtained by extrapolating the plots in the range of 9 mm $\leq L$ was also given in Table I as fracture toughness. It must be noted that thickness is replaced by basis weight [6] in our study, too.

3.2. Instrumentation

The fracture toughness testings were made with a pair of line-type clamps (PAPRICAN Special Clamps) mounted on an Instron type tensile testing machine (Shimadzu Auto-graph AGS-100) with a span distance of 100 mm and crosshead speed of 10 mm/min. The thermography system (NEC-San'ei Thermo-tracer 6T62) was set up to observe the notched area and the

TABLE I Basic properties of the samples

	UKP-sack		Machine glazed		Newsprint		Filter	
	MD	CD	MD	CD	MD	CD	MD	CD
Basis weight, g/m^2	47	47	41	41	40	40	100	100
Thickness, μm	70	70	55	55	66	66	174	174
Sheet density, kg/m^3	670	670	745	745	611	611	575	575
Tensile index, Nm/g	79.4	33.9	83.3	25.6	75.0	22.6	29.0	18.9
Elongation at failure, %	1.0	3.0	1.8	2.7	1.5	2.2	1.7	2.9
Elastic modulus, kNm/g	12.4	3.5	10.9	3.5	9.1	2.3	5.3	3.0
Fracture toughness[a] (w_e), Jm/kg	27.8	24.1	15.0	11.4	17.3	13.5	10.1	7.8

[a]Estimated by the EWF method (ligament length range of 9 mm $\leq L \leq$ 21 mm).

surrounds with a close-up lens. Details of the experimental conditions were the same as those described in the previous paper [5]. All testings were made at the standard atmosphere.

4. Results and discussion

4.1. Development of the plastic deformation zone during the testing

The developing pattern of the plastic deformation zone under straining varied with sample and ligament length. However, they were able to be classified into three types: 1. type (i)—appearing through whole the ligament in a vague manner and developing into a circular (or oval) zone; 2. type (ii)—appearing from notch tip and amalgamating into a circular (or oval) zone; and 3. type (iii)—appearing from notch tip and not amalgamating into a circular (or oval) zone.

Typical examples of these types are shown in Figs 2–4 as a series of successive close-up temperature distribution images, and the corresponding load-displacement relationships are given in Figs 5–7, respectively. Every initial location of the specimen with notches is superimposed with white lines on temperature distribution image (Figs 2a, 3a, 4a). Temperature scale is graduated by temperature rise compared with initial average temperature of specimen.

The case of UKP-sack paper (CD/$L = 5$ mm) is shown as the example of type (i). In this case, at halfway through the plastic deformation region in the load-displacement curve, the blue colored spots whose temperature is higher than that of the surroundings begin to appear through whole the ligament region in a vague manner (Fig. 2c), and increase their intensity with increase of the displacements. At the maximum load point (Fig. 2d), these spots are confined in a circular zone. After that point, higher temperature zones as indicated by green or yellow color begin to appear at around the notch tips (Fig. 2e) and extend toward the inside (Fig. 2f, g) until the final sheet failure. This means that the circular (oval) zone is attained before or at crack initiation, i.e. maximum load point [5].

The case of UKP-sack paper (CD/$L = 13$ mm) is shown as the example of type (ii). In this case, the blue colored spots appear at around the notch tips (Fig. 3b) in a concentrated manner and begin to increase their area. Until the maximum load point, they keep their color uniform and they are confined within clear-cut shapes at the notch tips (Fig. 3c,d). After that point, these two spots begin to extend toward inside (Fig. 3e) and join together (Fig. 3f) to make a circular (oval) zone. And at the same time, temperature rise at the notch tips (as indicated by green or yellow color), which means crack propagation, are observed too. This means that the circular (oval) zone is attained after crack initiation as described in the previous paper [5].

The case of UKP-sack paper (MD/$L = 5$ mm) is shown as the example of type (iii). In this case, the blue colored spots appear around the notch tips in a concentrated manner, too (Fig. 4e) and begin to increase their area (Fig. 4f,g). Their way of developing until the maximum load point is the same as those in type (ii). But the final sheet failure occurs soon after the maximum load point (Fig. 4h), so that they do not show an amalgamated zone at all.

In terms of the way of the appearance, type (ii) and type (iii) are essentially the same and they are different from type (i). The difference between type (ii) and type (iii) only depends on the length of the period after the maximum load point till the final sheet failure.

Plastic deformation zone appearance for all specimens were classified and are shown in Table II. The types vary with ligament length, tensile direction, and kind of papers. Specimens with small L are likely to belong to type (i), while those with large L are likely to belong to type (iii). CD specimens have the larger upper limit of L for type (i) than MD specimens; 5 or 4 mm for CD, 3 or 2 mm for MD. Specimens with the medium condition between type (i) and type (iii) seem to belong to type (ii).

Similar results showing the type (i) deformation for the specimen with small L were also observed in the same handsheet used in the previous paper [5].

As mentioned before, the EWF method is valid if there is complete yielding of the ligament before crack initiation, i.e. maximum load point. And its shape is theoretically assumed to be circular on the ligament (see Fig. 1). Taking the matter into consideration, only the type (i) specimen shows circular (or oval) plastic deformation zone before or at the maximum load point and fulfills the original assumption of the EWF method best among these types. While the plastic deformation zone of type (ii) before or at the maximum load point is small and not circular and that of type (iii) is very small and far from circular. Furthermore, size ratio of the observed plastic deformation zone before or at the maximum load point to the theoretical circular plastic deformation zone as shown in Fig. 1 decreases with an increase of L. These results demonstrate that the specific essential work of fracture can be determined as

TABLE II Type of the plastic deformation zone appearance for all specimens

		$L = 1$ mm	$L = 2$ mm	$L = 3$ mm	$L = 4$ mm	$L = 5$ mm	$L = 9$ mm	$L = 13$ mm	$L = 17$ mm	$L = 21$ mm
UKP-sack	MD	(i)	(i)	(ii)	(iii)	(iii)	(iii)	(iii)	(iii)	(iii)
	CD	(i)	(i)	(i)	(i)	(i)	(ii)	(ii)	(ii)	(ii)
Machine glazed	MD	(i)	(i)	(i)	(ii)	(iii)	(iii)	(iii)	(iii)	(iii)
	CD	(i)	(i)	(i)	(i)	(ii)	(ii)	(ii)	(ii)	(ii)
Newsprint	MD	(i)	(i)	(ii)	(iii)	(iii)	(iii)	(iii)	(iii)	(iii)
	CD	(i)	(i)	(i)	(i)	(ii)	(iii)	(iii)	(iii)	(iii)
Filter	MD	(i)	(i)	(i)	(i)	(ii)	(iii)	(iii)	(iii)	(iii)
	CD	(i)	(i)	(i)	(i)	(i)	(ii)	(iii)	(iii)	(iii)

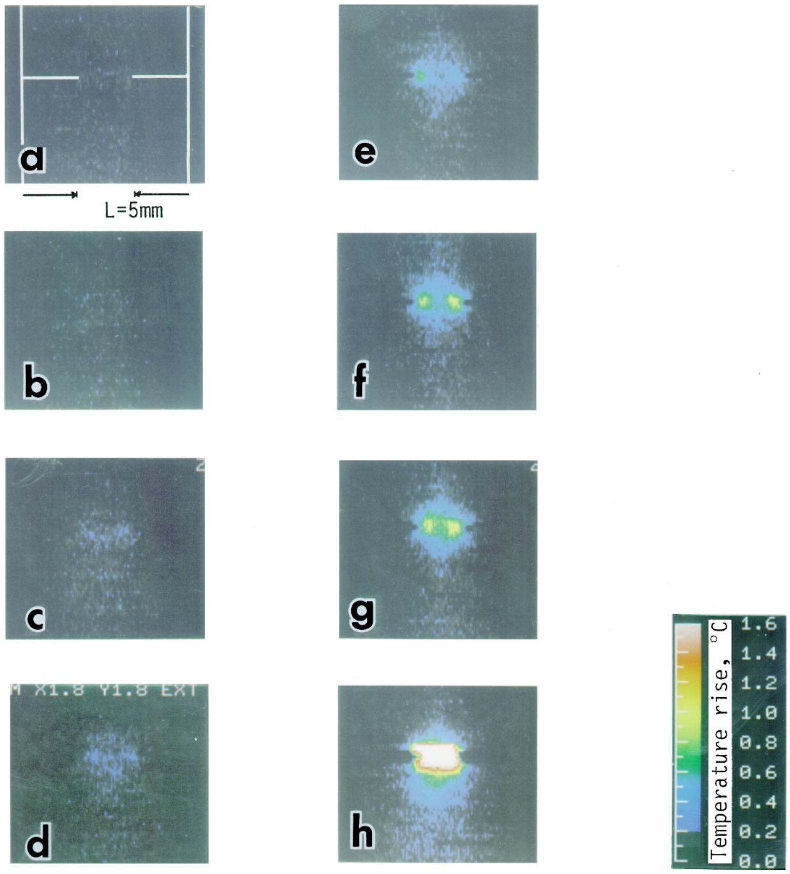

Figure 2 A series of close-up temperature distribution images of the UKP-sack paper (CD/L: 5 mm); (a)–(h) correspond to the positions on the load-displacement curves in Fig. 5.

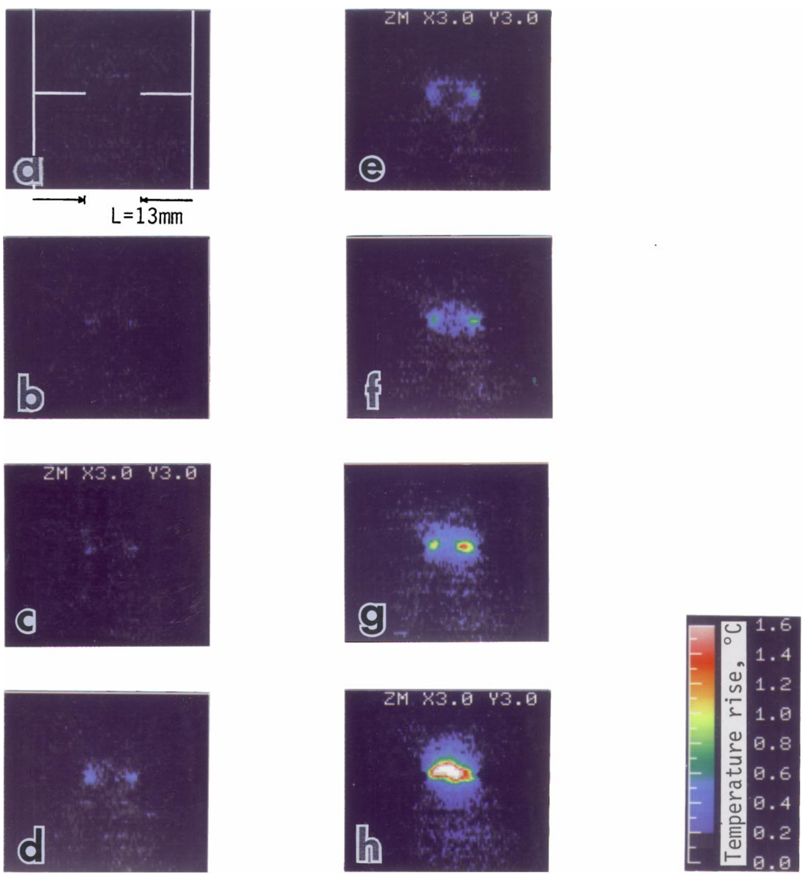

Figure 3 A series of close-up temperature distribution images of the UKP-sack paper (CD/L: 13 mm); (a)–(h) correspond to the positions on the load-displacement curves in Fig. 6.

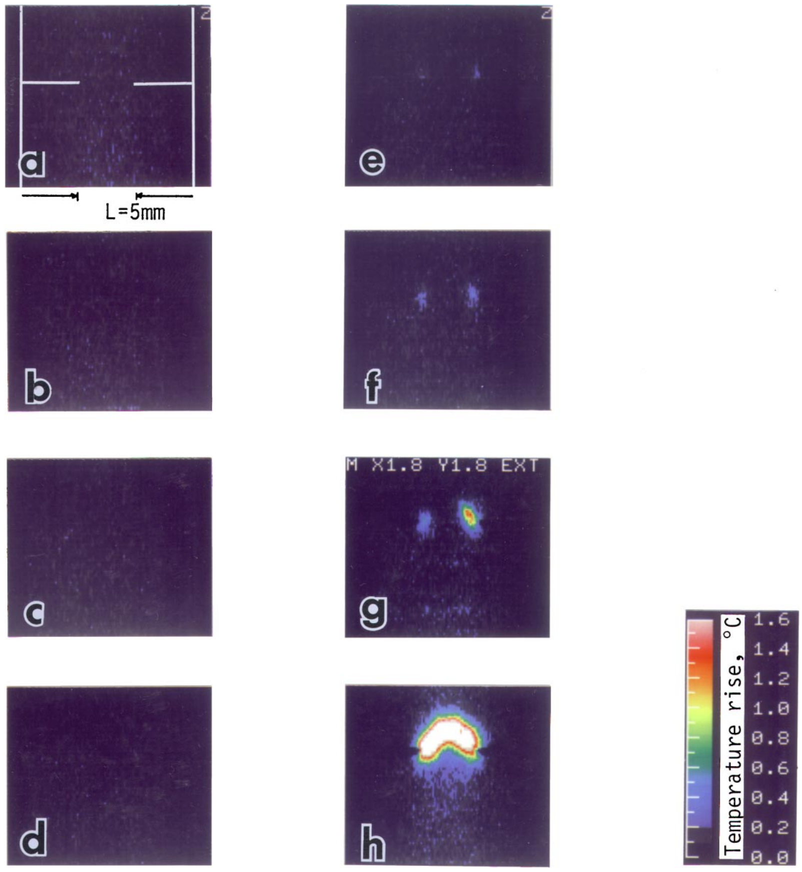

Figure 4 A series of close-up temperature distribution images of the UKP-sack paper (MD/L: 5 mm); (a)–(h) correspond to the positions on the load-displacement curves in Fig. 7.

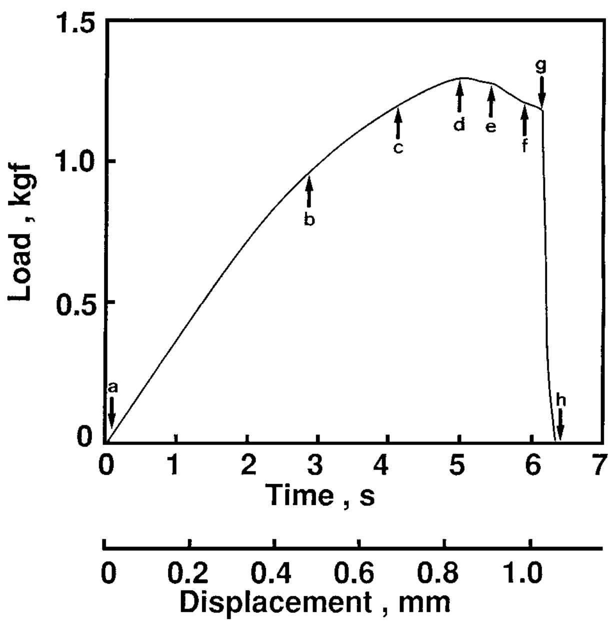

Figure 5 Load-displacement curve for the UKP-sack paper (CD/L: 5 mm).

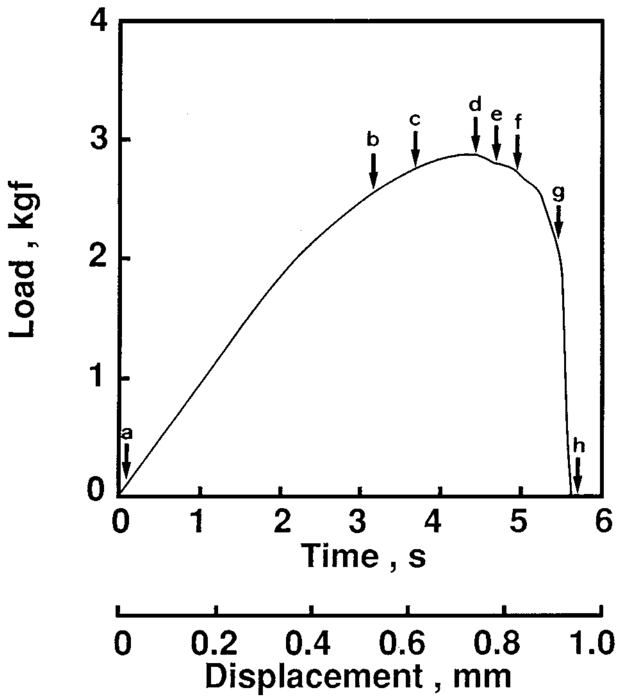

Figure 6 Load-displacement curve for the UKP-sack paper (CD/L: 13 mm).

proposed by Cotterell and Reddel [8, 9], only in the case of extremely small L where type (i) deformation occurs.

4.2. Experimental plots for the estimation of fracture toughness

Fracture toughness by the EWF method, w_e, is estimated by extrapolating linear L-w_f plots to $L = 0$. Straight lines were obtained from these linear plots by the method of least squares.

Some examples of L-w_f plots are shown in Fig. 8 for UKP-sack paper (MD), Fig. 9 for UKP-sack paper (CD), Fig. 10 for filter paper (MD), and Fig. 11 for filter

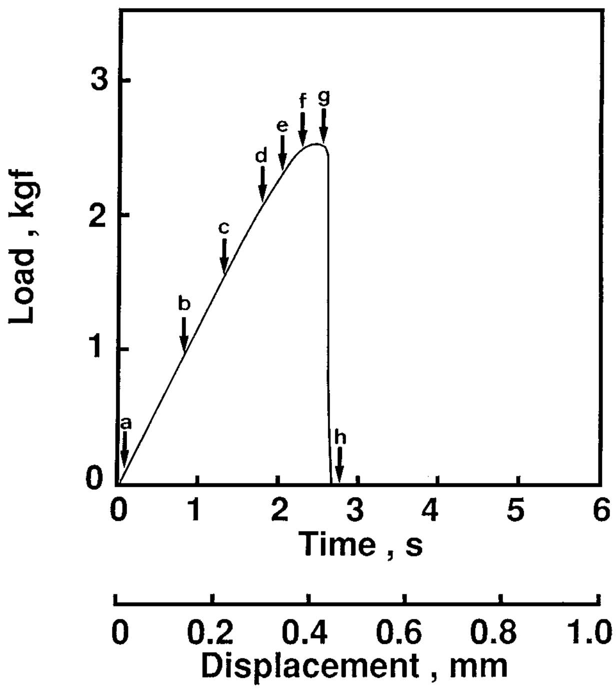

Figure 7 Load-displacement curve for the UKP-sack paper (MD/L: 5 mm).

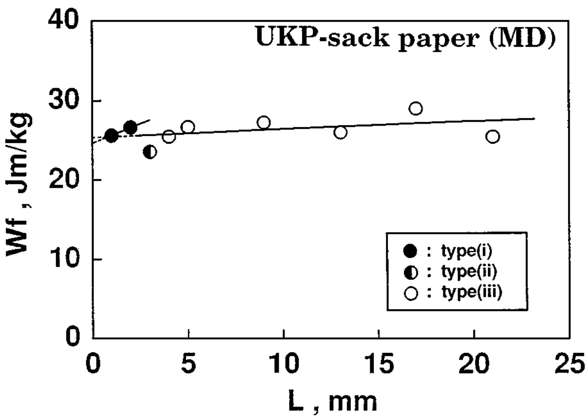

Figure 8 Experimental plots between work of fracture (w_f) and ligament lengths (L) for UKP-sack paper (MD).

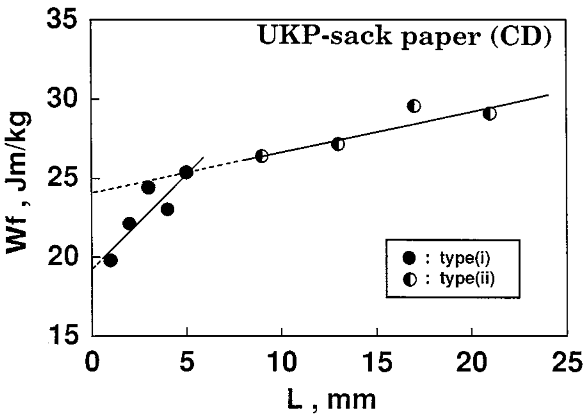

Figure 9 Experimental plots between work of fracture (w_f) and ligament lengths (L) for UKP-sack paper (CD).

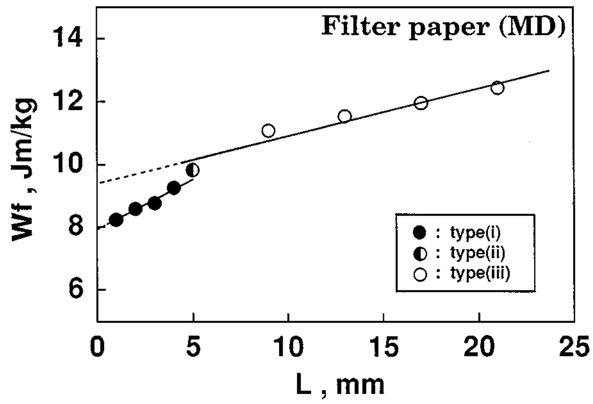

Figure 10 Experimental plots between work of fracture (w_f) and ligament lengths (L) for filter paper (MD).

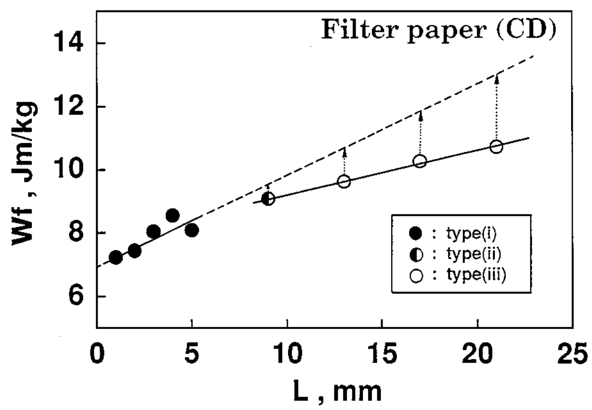

Figure 11 Experimental plots between work of fracture (w_f) and ligament lengths (L) for filter paper (CD). Dotted line arrows show the correction considering the size ratio of plastic deformation zone.

paper (CD). Every spot is an average of at least five results. Although some papers show little difference in the linear relations between type (i) group and type (ii) & (iii) group (Fig. 8), many papers show two different linear relations for type (i) group and type (ii) & (iii) group (Figs. 9–11). Considering that type (i) specimens satisfy the original assumption of EWF method, w_e obtained from type (i) plots is correct as mentioned before.

In the estimation of w_e, linear L-w_f plots using Equation 3 assumes that shape factor of the plastic deformation zone β is constant. However, the plastic deformation zone in type (ii) & (iii) deformation at the maximum load point is smaller than that in type (i) deformation as described before. Therefore, if the shape factor β is a size ratio of the observed plastic deformation zone to the theoretical circular plastic deformation zone on the ligament as shown in Fig. 1, and also a correction to keep β constant is made, the plots of type (ii) & (iii) have to move upward as shown by dotted line arrows in Fig. 11. The corrected data may be on the extension (broken line in Fig. 11) of the linear relation from type (i) plots.

5. Concluding remarks

The plastic deformation zone observed by thermography at the maximum load point is generally small

TABLE III Type of the plastic deformation zone appearance for UKP-sack paper. (Effect of specimen dimension and its comparison with effect of ligament length)

	L mm	Width mm	Span length mm	Plastic deformation zone appearance	Span length mm	Plastic deformation zone appearance
• Specimen dimension (span length : width = 100 : 6)						
UKP-MD	2	6	100	type (i)	100	type (i)
	3	9	150	type (iii)	100	type (ii)
	4	12	200	type (iii)	100	type (iii)
	2	6	100	type (i)	100	type (i)
UKP-CD	3	9	150	type (i)	100	type (i)
	4	12	200	type (i)	100	type (i)
• Specimen dimension (span length : width = 100 : 15)						
UKP-MD	5	15	100	type (iii)	100	type (iii)
	13	39	260	type (iii)	100	type (iii)
UKP-CD	5	15	100	type (i)	100	type (i)
	13	39	260	type (ii)	100	type (ii)

and far from the theoretical circular plastic deformation zone on ligament which is the original prerequisite of the EWF method except for the specimens having extremely small L. Thus, the application of EWF method to paper materials should be limited on the specimens having extremely small L. Although the specific essential work of fracture can be determined using the data from these specimens, great care is needed for specimen preparation and the testing. The specific essential work of fracture obtained is less than that from ordinary size specimens. Furthermore, the results suggest that J-integral which assumes a plastic deformation at the notch tip, on the contrary, is not applicable for the specimens having extremely small L.

6. Appendix: effect of specimen dimension

As Seth pointed out, the specific essential work of fracture has been thought to be independent of specimen dimension [3], on the other hand Yu and Kärenlampi recently showed that the specimen dimensions (length/width and crack length/width ratios) affected the work of fracture [10]. In our study specimen width was varied under constant length, i.e. specimen dimension (length/width ratio) was not the same. Therefore, it becomes necessary to clarify whether the type of plastic deformation zone appearance depends on the ligament length or the specimen dimension. Some specimens of UKP-sack paper with different ligament length and constant specimen dimension were further studied. Sizes of the specimen and the resulting type of plastic deformation zone appearance are shown in Table III. Although the result does not cover the wide range of samples and the specimen dimension, same ligament length shows the same type of plastic deformation zone appearance irrespective of specimen dimension. Thus, the type depends more on L than on specimen dimension.

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